Counting, measuring and the semantics of classifiers
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Central claim: the mass/count distinction between two types of nominals has its direct correlate at the level of classifier phrases: Classifier phrases like two bottles/litres of wine are ambiguous between a counting or individuating reading and a measure reading. On the counting reading, this phrase has count semantics, on the measure reading it has mass semantics.

Outline of talk:
I. Measure and counting readings of classifiers have different syntactic structures and different compositional interpretations
II. A consequence of (I) is that measure classifier phrases should have the semantics of mass expressions while counting classifier phrases should have the semantics of count expressions. We show that there is good empirical evidence to support this claim.
III. We give a semantic analysis of the interpretation of classifier expressions in the framework of Rothstein (2010), in which the measure/count interpretations of classifier phrases differ analogously to the mass/count distinction at the NP and DP levels.

PART I: Individuating vs measure readings of classifier constructions

1.1 Data
In typical mass/count languages, numeral modifiers modify count nouns directly. In many languages, with numerals greater than one the nominal is marked as plural as in (1):

(1) three flowers/four books/*three flour(s) .

Classifiers like box of N, cup of N are used to count mass nouns (2):

(2) *three flours vs three cups of flour

Measure expressions may also be used to count mass nouns (3):

(3) three kilos of flour

Quantities of plural nouns can be counted (4) as in where the classifier is used to ‘repackage” pluralities into higher order entities which can then be counted.

(4) three boxes of books, three kilos of books
Observation: (Doetjes 1997, Chierchia 1998, Landman 2004 and others) classifier phrases like two glasses of water are ambiguous between an ‘individuating’ reading (5a) and a ‘measure’ reading (5b):

(5) a. Mary, bring two glasses of water for our guests!
   b. Add two glasses of water to the soup!

The measure reading is equivalent to the explicit N+ful classifier:

(6) a. Add two cup(ful)s of wine to the soup.
   b. Bring two cup(#ful)s of wine for our guests.

Distribution is possible over individual/counting readings but not measure readings:

(7) The two glasses of wine(#in this soup) cost 2 Euros each.

(Other differences e.g. relative clause formation, number agreement, anaphoric dependence, discussed in Rothstein 2009.)

1.2 Analysis: based on Landman 2004:
On the individuating reading: two glasses of wine denotes actual glasses containing wine. On the measure reading: two glasses of wine denotes wine to the measure two glasses. So, in the individuating reading, glasses is the nominal head of the phrase, as in (8)

(8) 

\[
\text{DP} \\
\text{D} \text{three}_i \\
\text{NumP} \\
\text{NUM} \text{t}_i \\
\text{NP} \\
\text{N} \text{cups} \text{(of)} \text{DP} \text{wine}
\]

Following Landman (2003, 2004) we assume that the numeral is essentially adjectival, and begins in NUM, raising to the determiner in argument position if the determiner phrase is empty. Landman (2003) shows that if the determiner is filled, and the adjective does not need to raise, permutation with other adjectives is possible:

(9) We sent the ferocious three lions to Blijdorp and kept the mild three lions at Artis.

In the measure reading glasses is a modifier analogous to explicit measure phrases such as kilo of type <n, <e,t>>. These combines first with the numeral three (which does not raise). The complex modifier then applies to the nominal head wine (10):
In both cases, *of*-insertion is a late phenomenon satisfying surface constraints.

These structures offer the following interpretations (first versions):
We assume that the bare noun denotes a kind (Carlson 1977. Chierchia 1998), and shifts to a predicate interpretation via the $\cup$ operation, which maps kinds onto the set of (singular and plural) entities which instantiate the kind

(i) interpretation of individuating classifier phrases based on (8)

$[[\text{glass}]] = \text{GLASS}$

$\text{SHIFT}([[\text{glass}]]) = \lambda y \lambda x. x \in \text{GLASS} \land \text{CONTAIN}(x, y)$

$[[\text{glasses of wine}]] = \lambda x. x \in \text{GLASSES} \land \text{CONTAIN}(x, \text{WINE})$

$[[\text{three glasses of wine}]] = \lambda x. x \in \text{GLASSES} \land \text{CONTAIN}(x, \text{WINE}) \land \text{CARD}(x) = 3$

(ii) interpretation of measure classifier phrases based on (8)

*Glass* is a measure expression, of type $<n, <e, t>>$

It combines first with a numeral to form a predicate and then applies to a predicate nominal head via standard modification operations.

The operation which turns *glass* from a nominal to a measure expression is introduced either explicitly by $-\text{ful}$ or by a null correlate of $-\text{ful}$.

$[[\text{Glasses(ful)}]] = \lambda n \lambda x. \text{MEAS}(x) = <n, \text{GLASS}>$

$[[\text{three glasses}]] = \lambda x. \text{MEAS}(x) = <3, \text{GLASS}>$

$[[\text{three glasses of wine}]] = \lambda x. x \in \sim \text{WINE} \land \text{MEAS}(x) = <3, \text{GLASS}-\text{FUL}>$

1.3. Syntactic support for this analysis

(i) When the classifier is a nominal head and the number is a determiner, an adjectival modifier should be able to come between the number and the classifier, as in (11a). When Num + N is a measure predicate, an adjective should not intervene between them as in (11b).

(11) a. The waiter brought three expensive glasses of cognac.

   b. *She added three expensive glasses(ful) of cognac to the sauce*

(ii) Conversely, only when Num + N is a measure predicate, can it scope under another adjective. In an individuating construction this isn’t possible:
(12) a. You drank/spilled an expensive three glasses of wine!
   b. The waiter brought an expensive three glasses of wine!
   c. An expensive ten seconds of silence on the international telephone line followed.
   (Sarah Caudwell: Thus was Adonis Murdered)

(iii) Crosslinguistic support: Modern Hebrew (Rothstein 2009)

Indefinite construct state classifier constructions are ambiguous between measure and counting reading:

(13) **arba'im ve-štaim kufsaot sefarim**
    lo nixnasot la-madafim šelanu
    forty and two boxes books no enter(f.pl) to-shelves of-us

"Forty-two boxes of books don't fit on our shelves".

*Measure reading*: the books don’t fit on our shelves.
*Individuating (counting) reading*: the boxes don’t fit on our shelves (e.g. in an archive)

The construct state is a “syntactic word” formed out of two bare Nouns (Borer 1999, 2008): The relation between the two Ns is only partially constrained:

\[
[N_1 \quad N_2] \text{ can be analysed}
\]

(i) with \(N_1\) as the head and \(N_2\) as the complement - this is the individuating reading

(ii) with \(N_2\) as the head and \(N_1\) modifying the head - this is the measure reading.

**Individuating reading**: \([42 \; \text{kufsaot} \text{HEAD} \; \text{sefarim} \text{COMPLEMENT}]\]

i.e. kufsaot ‘boxes’ is the head, sefarim ‘books’ is the complement and 42 modifies the phrase.

**Measure reading**: \([42 \; \text{kufsaot} \text{MODIFIER} \; \text{sefarim} \text{HEAD}]\]

42 and kufsaot combine to form a complex predicate and modify sefarim ‘books’

**Prediction**: if the complex predicate cannot be constructed, the measure reading is impossible: This prediction is born out in two ways:

**Prediction 1**: Definite numerical construct state constructions do not allow measure readings:
(14a) is the indefinite construct state analogous to (13). It has the same two possible syntactic analyses and is ambiguous between measure and individuating reading

(14a) \([šloša bakbukey yayin]\)

    – \([šloša \; \text{bakbukey} \text{HEAD} \; \text{yayin} \text{COMPLEMENT}]\) (individuating reading)

    OR \([ [šloša \; \text{bakbukey} \text{MODIFIER}] \; \text{yayin} \text{HEAD}]\) (measure reading)

(14b) is an definite construct state. It is the only way to express ‘the three bottles of wine’:

(14b) \([šlošet \; \text{bakbukey} \text{HEAD} \; \text{ha-yayin} \text{COMPLEMENT}]\text{NP}\]

three bottles DEF-yayin

"The three bottles of water"

šlošet ‘three’ is a construct state head; bakbukey yayin ‘bottles of wine’ is a construct state phrase, and must be analysed as a constituent, the complement of šlošet, as in (14b). So bakbukey cannot be construed with šlošet and thus cannot form a measure modifier.
As predicted, only the individuating and not the measure reading of (14) is available.

(15) gives a context in which the desired definite measure reading is impossible in Modern Hebrew, but possible in English:

(15)  hizmanti esrim orxim ve- hexanti esrim ka’arot marak be- sir gadol.
I invited twenty guests and I prepared twenty bowls soup in- pot big
“I invited twenty guests and I prepared twenty bowls of soup in a big pot”

rak šiva-asar orxim higiu, ve- nišar marak le-šloša anashim.
only seventeen guests came, and was left soup for three people.

#šaloš ka’arot ha- marak (ha- axaronot) nišaru b- a- sir.
three bowls DEF soup DEF last remained in DEF pot

Intended but impossible reading: “Only 17 guests arrived, and enough soup was left for three people. The (last) three bowls of soup remained in the pot.”

Prediction 2: definite measure constructions are ungrammatical:
If a definite construct state nominal does not allow a measure reading syntactically but the content of the construct state only allows a measure reading semantically, then we will get conflict between syntax and semantics which will result in an ungrammatical construction. Thus, indefinite construct state constructions are possible with measure heads such as kilo as in (16a), but the definite forms are not grammatical.

(16) a. xamiša kilo kemax
   5 kilo kemax
   “five kilos of flour”

b. *xamešet kilo ha- kemax
   five kilo DEF flour
   intended reading: “the five kilos of flour”

Conclusion: there is evidence in support of the syntactic structures in (8) and (10).

Part II. Classifier phrases and the mass/count distinction
2.1 The prediction:
The analysis in Part I makes the following prediction:

If the measure phrase two kilos(of) or two glasses(of) is an intersective predicate and modifies a mass expression as in two kilos of flour/two glasses of water, then the whole expression should be of the same type i.e. mass.

If the expression two glasses of water is an individuating expression, with the count nominal glasses (of) as its head, then the whole expression should be a count expression.

This is intuitively right: measure phrases give properties of quantities (denotations of mass expressions). As an individuating classifier, nominal expressions such as glasses of N pick out individual entities containing N, and these entities can be counted.
In short: Measure readings of *two glasses/litres of water* are mass expressions. Individuating readings of *two glasses of water* are count expressions.

In the rest of this section, we show that there is good evidence to support this claim. (Note that we are restricting our attention to “container” classifiers)

2.2. measure readings vs individual readings of e.g. two glasses of milk

Tests for count vs mass at the nominal level:
(i) modification of N by numerals:
(17) a. three flowers; *three flour(s)

(ii) pluralisation and agreement:
(17) b. the flowers are on the table; the flour is/*are on the table

(iii) sensitivity of determiners e.g. many/much to the mass/count distinction; this shows up in nominals (17c) and partitives (17d):
(17) c. many/*much flowers; much/*many flour
(17) d. three/*many of the flowers; much/*three of the flour

(vi) reciprocal resolution (Gillon 1992) and other distributive phenomenon:
(17) e. The carpets and the curtains resemble each other (ambiguous).
(17) f. The carpeting and the curtaining resemble each other (unambiguous).

In (17e), the maximal sums or their atomic parts can be antecedents for the reciprocal; in (17f), only the maximal sums can be antecedents for the reciprocal.

We apply these tests to *the NUM bottles of wine*: where relevant, measure readings patterns with mass nouns and counting readings patterns with count nouns: (note that (i), direct modification by numerals, is not relevant.)

ii. pluralisation and agreement: Pluralisation does not distinguish between the readings. But agreement does: in the counting reading, where the plural count classifier is the lexical head of the phrase, the verb must be plural, (18a). In the measure reading, where the classifier has shifted to a modifier taking a plural number argument, the mass noun is head of the phrase and the verb may be (and is possibly preferred to be) singular, (18b):

(18) a. The two bottles of wine that we carried here were/#was heavy. (C)
    b. The two teaspoons/50 mililitres of wine we added to the sauce gives/
   ?give it an extra flavour. (M)

Dutch (Doetjes 1997): true measure predicates such as liter with a plural number argument do not take number agreement (note this does not hold of nominal classifiers shifted to a measure reading such as fles) Measure predicates can shift to an individualised quantity reading, in which case they agree with the number:
(19) a. Ik heb twintig liter frisdrank bezorgd voor het feestje.
    I have 20 liter soft-drink delivered for the party.
    “I have delivered 20 liters of soft-drinks for the party”.

    b. Ik heb twintig liter-pl frisdrank bezorgd voor het feestje.
    Preferred reading: “I have delivered 20 liter-bottles of drink for the party.”

(20) a. Twintig liter water staat (sg) in de kelder.
    20 litre water stand in the basement (Measure reading only)

    b. Twintig liters water staan (pl) in de kelder
    20 litres water stand in the basement (Individualised litre bottles reading only)

iii. sensitivity of determiners to the mass/count distinction: many/much
    When the Classifier Phrase is interpreted as a counting expression with the classifier as head of
    the phrase, the CIP can be modified by many. On the measure reading, the modifier must be
    much (21d). Note the verbal agreement with many is plural and with much is singular, in

(21) a. Not many of the twenty bottles of wine that we bought were drunk/opened.  (C)
    b. Not much of the twenty bottles of wine that we bought was drunk.  (M)
    c. Not much of the twenty bottles of wine that we bought was #opened.  (M)
    d. I have used much/*many of the ten kilos of flour that there was in the cupboard.

    Note that in order to be interpreted as a measure modifier, the classifier must apply first to a
    number argument. In the absence of an number, the measure reading is not (usually) available:
    (22b) contrasts with (21b):

(22) a. Not many of the bottles of wine that we bought were drunk/opened.
    b. #Not much of the bottles/litres of wine that we bought was drunk.  (M)

    Explanation of (22): in the measure reading, the measure word combines first with a
    number to form a complex predicate. Without a number word, the measure reading is
    dispreferred. So (22a) has the count reading easily. (22b) is infelicitous since it is a measure
    reading context, but the lack of number in the classifier makes the measure reading
    dispreferred.

    Note though that we can get numerical partitives in each case. We return to these below.

(23) a. Three of the bottles of wine that we bought were opened. (C)
    b. About three of the six bottles/litres of wine that we bought was drunk (all in all). (M)

iv. reciprocal resolution and other distributive phenomena.
    Atomic parts of individuating classifier denotations are antecedents for reciprocals.
    Measure readings do not provide such atomic parts as antecedents for reciprocals.

(24) a. The cook mixed three kilo packs of flour with each other.
    b. #The cook mixed three kilos of flour with each other.
(25) a. The twenty bottles of wine and the twenty bottles of beer we had not yet opened stood next to each other on the shelf. (C) (No constraints on how the beer bottles and the wine bottles are arranged.)

b. The twenty liters of wine and the twenty liters of beer that we bought stood next to each other in the cellar. (M) (Preferred reading: The beer is standing next to the wine.)

In Dutch, this shows up even more clearly:

(26) a. De vijftien liters melk en de vijftien liters jus d'orange liggen op elkaar gestapeld in de kelder.

“the 15 litres of milk and the 15 litres of orange juice are stacked on top of each other in the basement”.

Either the 30 individual litre packs are stacked on each other OR two containers of 15 litres are stacked on each other.

(26) b. De vijftien liter melk en de vijftien liter jus d'orange liggen op elkaar gestapeld in de kelder.

“The 15 litres of milk and the 15 litres of orange juice are stacked on top of each other in the basement”.

ONLY READING: 2 containers of 15 litres are stacked on top of each other.

When the reading is unambiguously measure, the antecedent of the reciprocal is the pair of two (maximal) quantities. On the individuating reading, the individual litre containers i.e. the atomic parts of the maximal entities can also constitute the antecedent for the reciprocal.

Conclusion:
- when a mass noun is the complement of a measure classifier, and the Classifier Phrase has the properties of a mass nominal.
- when a mass noun is the complement of an individuating classifier, the Classifier Phrase has the properties of a count nominal.

2.3 measure readings vs individual readings of e.g. two boxes/kilos of books

A more complicated version of the same issue:

In the previous section, the measure expression e.g. two glasses/two litres did not change the mass status of the complement wine. What happens when the complement of the classifier is not a mass noun, but a bare plural count noun?

(27) a. When we left for the Netherlands we sent 16 kilos of books.

b. When we left for the Netherlands we sent four boxes of books.
Prediction:
a. if measure phrases consistently modify mass nouns, then the measure phrase 16 kilos, and also four boxes on its measure reading, should modify a mass noun.

b. This means that in (27a) books should be a mass noun modified by 16 kilos and that (27b) should be ambiguous between
   (i) the counting reading when boxes is the nominal head of the phrase. On this reading four boxes of books has the interpretation of a count noun.
   (ii) the measure reading when books should be mass and four boxes interpreted as a measure phrase which modifies it. On this reading four boxes of books has the interpretation of a (complex) mass noun.

- Is there evidence that books, 16 kilos of books and four boxes of books on its measure reading, behave like mass expressions. Yes, as we show in the rest of this section.
- Is there a natural way to derive the interpretation of books as a mass noun? Yes, as we show in the next section.
- What are the implications of saying that books is a mass noun? We discuss this at the end of section 3.

Evidence that books is a mass noun in measure readings of classifier constructions.
ii. pluralisation and agreement:
When the classifier phrase is individuating, the verb agreement must be plural (28a).

(28) a. The twenty boxes of books that we sent were/*was in the study.

When it is a quantity expressions, judgements vary. Singular agreement is possible (and even preferred) when the predicate is a quantity predicate, forcing an amount reading of the classifier phrase, (28b/c). It is also possible in (28d).

(28) b. The five boxes of books/twenty kilos of books that we sent was not enough to keep my daughter supplied with reading matter.
   c. The twenty boxes of books that we sent has kept my daughter supplied with reading matter for the whole year.
   d. Twenty kilos/boxes of books was/were put through the shredder that night.

In (28e) plural verbal agreement seems to be obligatory, but the only possible reading of the classifier phrase is individuating.

(28) e. The twenty boxes of books that we brought were/#was piled on the shelves.
   (the boxes were piled up)

Dutch: with the measure phrase kilo, singular or plural agreement is OK with a slight preference for singular. Predictably, the plural kilos is never possible, because books are individuated via their individual book-identities, and not by being put into kilo packages.

(29) a. Twintig kilo boeken werd/*werden door de papiervernietiger gemalen.
   20 kilo-sg books was-sg/?were-pl through the paper shredder ground
   “Twenty kilos of books was ground through the paper-shredder”
   b. # Twintig kilos boeken werd/werden door de papiervernietiger gemalen.
   20 kilo-plg books was-sg/?were-pl through the paper shredder ground
iii. **sensitivity of determiners to the mass/count distinction**

When the classifier phrase has an individuating reading, it can be embedded under the count determiner *many*. When it has an measure reading, it can be embedded under the mass determiner *much*. Note that *much* induces singular agreement.

(30)  a. I have read many of the twenty boxes of books that we sent: (C)  
    b. #I have read many of the twenty kilos of books that we sent (C)  
    c. I have(n’t yet) read much of the twenty boxes/twenty kilos of books in our house. (M)  
    d. Not much of the twenty boxes/kilos of books that we sent was left unread by the end of the year.  
    e. A little of the twenty boxes/kilos of oranges that we picked was/#were enough to satisfy our desire to eat citrus fruit.

Notice that the mass and count readings do not entail each other.

(31) I have read much/many of the twenty boxes of books that we sent.

If most of the boxes are small and have a few books in them, and only some of the boxes have a lot of books in them, then the *I have read much of the ten boxes of books* does not entail *I have read many of the ten boxes*, nor vice versa.

As above, note that in the absence of a number, the measure reading is not available, (except when a null number expression meaning ‘huge quantity’ is indicated via intonation).

(32)  a. I haven’t read many of the boxes of books that we sent.  
    b. #I haven’t read much of the boxes/kilos of books that we sent.

iv. **reciprocal resolution and other distributive phenomena.**

In English, reciprocals requires plural count nouns as antecedents, see (33):

(33)  a. The shoes bumped against each other in the suitcase.  
    b. #The footwear bumped against each other in the suitcase.

Individuating (i.e.count) classifier phrases, provide natural antecedent for the reciprocal. In *(34) each other takes boxes(of books)* as its antecedent. The complement nominal *books* is not available since it has been ‘repackaged’.

(34)  a. 42 boxes of books were piled on top of each other on the shelves. (Only: the boxes are on top of each other.)  
    b. #3 boxes of books were piled on top of each other on different shelves.

(34b) is infelicitous because you need at least 4 boxes to be piled on top of each other on different shelves. If the complement was available as a potential antecedent, an alternative felicitous reading should be OK.

In measure classifier phrases, *three kilos/boxes of books*, the classifier *boxes* cannot be the antecedent, since it is a measure predicate. The nominal complement *boxes* should determine the antecedent for the reciprocal, since it is head of the phrase.
However, it is a mass noun, and so not an appropriate antecedent for the reciprocal (in English).

So (34a) above had only the individuating reading.

(35a) is find, but (35b), is infelicitous, as the reciprocal has no grammatical antecedent.

(35) a. Twenty kilos of books are lying in a heap on the floor.
   b.#Twenty kilos of books are lying on top of each other on the floor.

(35c/d) further illustrates the same point:

(35) c. The twenty boxes of books are standing next to each other on the shelves. (Antecedent = the boxes)
   d. #The twenty kilos of books are standing next to each other in a row.

When twenty kilos can be interpreted as an individuating expression meaning “twenty kilo-packs”, the reciprocal can take this individuating expression as an antecedent.

(36) The twenty kilos of flour/the twenty kilo-packs of flour are standing next to each other in a row on the shelf.

In sum: twenty kilos of books and twenty boxes of books on its measure reading
   - do not provide antecedents for reciprocals,
   - can be the complements of mass quantifiers like much and a little.

This implies that they are mass expressions.

Two additional arguments in support of this claim:

(i) In recipe contexts, bare plural conjoin with mass nouns. Assuming conjunction between like types, this implies they are mass expressions

(37) a. Add cheese, chives and (ground) peppercorns.
   b. “Enjoy this creamy blend of cream cheese, caraway seed, chives, and dillweed…”

(ii) Measure classifers take only bare noun complements (38a/b), since these nouns are heads which are to be modified. Individuating readings take a wider range of complements, and can be modified by number. This is expected since since these complements are arguments of a relational noun head.

(38) a. On the floor were piled four kilos of (*ten) books.
   b. On the floor were piled four boxes of ten books.
      (ONLY the count reading: i.e. the boxes were piled up).
   c. I unpacked/#read three boxes of ten books (ONLY container reading)
PART 3: Analysis

3.1. The mass count distinction (based on Rothstein 2010, ms).

Rothstein (2010) argues for a typal distinction between mass nouns and count nouns. This is based on the following points:

a. mass nouns (e.g. stone/furniture) and count nouns (e.g. stones/pieces of furniture) get their denotations with respect to the same entities. (Chierchia 1998).

b. some mass nouns (e.g. furniture) are naturally atomic, i.e. denote plural sets which are de facto the closure under sum of a set of inherently indivisible entities. Rothstein (2010) calls these sets ‘naturally atomic’.

c. some count nouns (e.g. fence, wall, sequence) are not naturally atomic, and the set of atoms in their denotation is varies from context to context.

Rothstein (2010): counting is a context dependent operation which counts the entities which in a relevant context count as atomic entities. Count nouns are countable because they encode grammatically the context in which they denote sets of atoms (or pluralities of atoms).

This is expressed grammatically in the following way.

1. Nominals are interpreted with respect to a complete atomic Boolean algebra $M$. Intuitively, $M$ is the mass domain. $\sqcup_M$, the sum operation on $M$, is the complete Boolean join operation; $\sqcap_M$ is the part of relation on $M$.

We assume with Chierchia (1998) that the set of atoms $A$ of $M$ is not fully specified, vague. (Nothing rests on this choice of mass domain; we assume it for simplicity.)

2. All nouns are associated with an abstract root noun. The denotation of a root noun, $N_{\text{root}}$, is a subset of $M$, defined as follows:

For some set of atoms, $A_N \subseteq A$, $N_{\text{root}} = A_N$, where $A_N = \{m \in M : \exists Y \subseteq X : m = \sqcup_M Y\}$

Root nouns never appear as lexical items: (this is different from Rothstein, 2010).
- Mass nouns denote the kind associated with the root noun. (see 3).
- (Singular) count nouns denote the set of semantic atoms derived from the root noun (see 5).

3. Mass nouns denote $\sqcap_{N_{\text{root}}}$, i.e. the kind associated with $N_{\text{root}}$. Following Chierchia (1998), we assume that kinds are defined via the maximal entity in the denotation of $N_{\text{root}}$.

Kinds are functions from worlds/situations onto the maximal entity instantiating $N_{\text{root}}$ in that world/situation, i.e. For any $N_{\text{root}}$ and world/situation $s$: $\sqcap_{N_{\text{root}}} = \lambda w. \sqcup_M (N_{\text{root}}, w)$. We restrict ourselves to extensional contexts here and assume that the denotation of a kind term is $\sqcap_{N_{\text{root}}}(w_0)$ (with $w_0$ the world of evaluation). This means that we can assume that the denotation of kind terms is of type $d$.

**Definition 1:**

(i) the interpretation of $N_{\text{mass}}$ is $\text{MASS}(N_{\text{root}}) = \sqcap_{N_{\text{root}}}(w_0)$

(ii) $\sqcap$ is the function from kind(-extensions) to sets of individuals such that for every kind(-extension) $d(w_0)$: $\sqcap(d(w_0)) = \{x. x \sqcap_M d(w_0)\}$

**Fact:** for every root noun $N_{\text{root}}$: $\sqcap(\sqcap_{N_{\text{root}}}(w_0)) = N_{\text{root}}$
4. Count nouns differ from mass nouns because they allow direct grammatical counting. Counting is putting entities in one-to-one correspondence with the natural numbers and requires a contextually determined choice as to what counts as one entity. This choice of what counts as one entity is encoded the notion of (counting) context k, which intuitively collects together the entities which count as atoms in k.

**Definition 2:**

A context k is a set of objects from M, k ⊆ M, K is the set of all contexts. The set of count atoms determined by context k is the set \( A_k = \{<d,k> : d \in k\} \)

5. Singular count nouns are derived from root nouns by a count operation \( \text{COUNT}_k \) which applies to the root noun \( N_{\text{root}} \) and picks out the set of ordered pairs \( \{<d,k> : d \in N \cap k\} \), i.e. the set of entities in \( N_{\text{root}} \) which count as one in context k.

**Definition 3:**

For any \( X \subseteq M \): \( \text{COUNT}_k(X) = \{<d,k> : d \in X \cap k\} \)

The interpretation of a count noun \( N_{\text{count}} \) in context k is: \( \text{COUNT}_k(N_{\text{root}}) \).

6. Plural count nouns are derived by applying the standard plural operation \( * \) to the first projection of \( N_k \).

**Definition 4:**

Assume:
- \( \pi_1(N_k) = \{d : <d,k> \in N_k\} \)
- \( \pi_2(N_k) = k \)

In default context k: \( \text{PL}(N_{\text{count}}) = *N_k = \{<d,k> : d \in *\pi_1(N_k)\} \)

Examples:
- \( \text{[stone}_\text{mass}] = \text{MASS}([\text{STONE}_\text{root}] \)
- \( \text{[stone}_\text{count}] = \text{COUNT}_k([\text{STONE}_\text{root}] \)
  \( = \{<d,k> : d \in \text{STONE}_\text{root} \cap k\} \)

\( \text{stone}_\text{mass} \) denotes the kind in \( w_o \) \( \text{stone} \) of type \( d \) i.e. the maximal quantity of stone in \( w_o \)

\( \text{stone}_\text{count} \) denotes a set \( \{<d,k> : d \in \text{STONE}_\text{root} \cap k\} \) of type \( d \times k \) i.e. the set of indexed entities which count as one in context k.

### 3.2 Individuating (i.e. counting) classifier phrases.

*Two boxes of books/two boxes of sugar/three cups of water*

We assume the structure in (8) above:

```
      (8)      
     / \      
    DP    
   /   \     
  D  NumP 
 /     \     
 three_i  
        / \ 
       /   \ 
      NUM NP 
     /   \   
    t_i  boxes  (of) DP sugar
```

We use $y$ as variable of type $d$, $y$ as a variable of type $d \times k$, and $y$ as a general variable over both types (thus including kinds).

Basic meaning of box: $\text{BOX}_k = \{x: \pi_1(x) \in \text{BOX} \land \pi_2(x) = k\}$
(We omit the conjunct “$\pi_2(x) = k$” in what follows. It is not relevant since we are dealing only with simple container classifiers here, and these expressions also denote atoms in $k$.)

box is an individuating classifier. It takes as its complement either a mass noun or a count noun e.g. (39):

(39) Three boxes of sugar/books

We assume that box raises to a complement taking noun, which assigns a thematic role CONTAIN to a direct object (see Borschev and Partee 2004, Partee and Borschev in press)
The argument of CONTAIN is either a simple argument or a generalized quantifier.
Mass complements are interpreted as denoting kinds.

Its (plural) form: boxes (nominal head): $\lambda y. \lambda x. \pi_1(x) \in \text{*BOX} \land \text{CONTAIN}(\pi_1(x), y)$

**Three boxes of sugar:**
boxes of sugar: $\lambda x. \pi_1(x) \in \text{*BOX} \land \text{CONTAIN}(\pi_1(x), \text{^SUGAR}_{\text{root}})$
three (boxes of sugar):
$\lambda P. \lambda x. \pi_1(x) \in P \land \text{CARD}(\pi_1(x) = 3)$ ($\lambda x. \pi_1(x) \in \text{*BOX} \land \text{CONTAIN}(\pi_1(x), \text{^SUGAR}_{\text{root}}))$
$= \lambda x. \pi_1(x) \in \text{*BOX} \land \text{CONTAIN}(\pi_1(x), \text{^SUGAR}_{\text{root}}) \land \text{CARD}(\pi_1(x)) = 3$

**Three boxes of books:** This is interpreted in the same way under the assumption that bare plurals denote kinds too.
$\lambda x. \pi_1(x) \in \text{*BOX} \land \text{CONTAIN}(\pi_1(x), \text{^BOOKS}_k) \land \text{CARD}(\pi_1(x)) = 3$

Individuating classifiers with non-kind complements arguments are interpreted similarly.

Since the classifier box is a relational noun derived from the count noun box, the count status of the classifier phrase follows automatically.

**3.3 Measure classifier phrases**
We assume the structure in (10) and the interpretations given there:

(10) $\begin{array}{c}
\text{DP} \\
\mid \\
\text{NP} \\
\mid \\
\text{MeasP} \\
\mid \\
\text{N} \\
\mid \\
\text{NUM} \\
\text{three} \\
\mid \\
\text{N}_{\text{meas}} \\
\text{boxes (of)} \\
\text{sand} \\
\mid \\
\text{N}_{\text{meas}} \\
\text{kilos (of)} \\
\text{sand}
\end{array}$
(i) measure phrases with mass complements

(i.i) three kilos of sand
Kilo denotes an expression of type <n, <d,t>>: \( \lambda n \lambda x. \text{MEAS}(x) = <n, \text{KILO}> \)
three kilos: \( \lambda x. \text{MEAS}(x) = <3, \text{KILO}> \)
this shifts to the modifier type <<d,t>,<d,t>>: \( \lambda P \lambda x. x \in P \land \text{MEAS}(x) = <3, \text{KILO}> \)

three kilos of sand: \( \lambda x. x \in \bigcup \cap \text{SAND}_{\text{root}} \land \text{MEAS}(x) = <3, \text{KILO}> \)
\[ = \lambda x. x \in \text{SAND}_{\text{root}} \land \text{MEAS}(x) = <3, \text{KILO}> \]
i.e. instantions of the SAND kind that measure three kilos.

(i.ii) three boxes of sand
Root meaning of box is: BOX_{\text{root}}. The derived measure reading is at type <n, <d,t>>, and is derived from the root meaning. In English, the operation which turns box from a nominal to a measure expression is introduced either explicitly by –ful or by a null correlate of –ful.

\( \lambda n \lambda x. \text{MEAS}(x) = <n, \text{BOXFUL}> \)
i.e. box(-ful) combines first with a numeral to form a predicate and then shifts to the modifier reading to apply to a nominal head.

Agreement is morphological, and not a semantic reflection of a pluralisation operation.

box(ful) (measure expression): \( \lambda n \lambda x. \text{MEAS}(x) = <n, \text{BOXFUL}> \)
three boxes(ful): \( \lambda x. \text{MEAS}(x) = <3, \text{BOXFUL}> \)
three boxes(ful)_{\text{MOD}} \( \lambda P \lambda x. x \in P \land \text{MEAS}(x) = <3, \text{BOXFUL}> \)
three boxes(ful) of sand: \( \lambda x. x \in \bigcup \cap \text{SAND}_{\text{root}} \land \text{MEAS}(x) = <3, \text{BOXFUL}> \)
\[ = \lambda x. x \in \text{SAND}_{\text{root}} \land \text{MEAS}(x) = <3, \text{BOXFUL}> \]
Crucially: while relational nominals take arguments at type d (or type <<d,t>,t>), measure phrases modify mass noun predicates, i.e. expressions of type <d,t>.

(ii) measure phrases with bare plural complements: three boxes/kilos of books
When the complement of the measure phrase is a count noun, the count noun must shift from the count type to the mass type.

A plural count noun is a predicate of type <d\times k, t>.
books_k denotes \( \{ <x,k>: x \in *(\text{BOOK}_{\text{root}} \cap k) \} \)

Measure phrases modifier predicates at type <d,t>, so the plural count noun denotation shifts to this type.

This shift makes use of the \( \pi_1 \) function.
SHIFT_{\text{MEAS}} (\( \{ <x,k>: x \in *(\text{BOOK}_{\text{root}} \cap k) \} \)) =
\[ = \pi_1(\{ <x,k>: x \in *(\text{BOOK}_{\text{root}} \cap k) \}) \]
\[ = *(\text{BOOK}_{\text{root}} \cap k) \]
This is an expression which can be directly modified by a measure predicate.
(ii.i) three kilos of books
kil: \( \lambda n \lambda x. \text{MEAS}(x) = \langle n, \text{KILO} \rangle \)
three kilos: \( \lambda x. \text{MEAS}(x) = \langle 3, \text{KILO} \rangle \)
three kilos\text{\small{MODIFIER}}: \( \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{KILO} \rangle \)

Three kilos of books:
\[ \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{KILO} \rangle \] [SHIFT(BOOKS_k)]
\[ = \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{KILO} \rangle \ast (\text{BOOK}_\text{root} \cap k) \]
\[ = \lambda x. x \ast (\text{BOOK}_\text{root} \cap k) \land \text{MEAS}(x) = \langle 3, \text{KILO} \rangle \]

(ii.ii) three boxes of books
Box(ful): \( \lambda n \lambda x. \text{MEAS}(x) = \langle n, \text{BOXFUL} \rangle \)
three boxes(ful): \( \lambda x. \text{MEAS}(x) = \langle 3, \text{BOXFUL} \rangle \)
three boxes(ful)\text{MODIFIER} \( \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{BOXFUL} \rangle \)

three boxes of books:
\[ \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{BOXFUL} \rangle \] [SHIFT(BOOKS_k)]
\[ = \lambda P \lambda x. x \in P \land \text{MEAS}(x) = \langle 3, \text{BOXFUL} \rangle \ast (\text{BOOK}_\text{root} \cap k) \]
\[ = \lambda x. x \ast (\text{BOOK}_\text{root} \cap k) \land \text{MEAS}(x) = \langle 3, \text{BOXFUL} \rangle \]

3.4 What does this analysis mean?

**Counting** is an context dependent operation which puts entities which count as atoms in the relevant context in one-to-one correspondence with the natural numbers.

(NB The denotation of e.g. *three* given above is simplified: the complete definition encodes the context dependence of the counting operation:

*Three* denotes a function from count noun denotations into count noun denotations and is of type \( \langle \langle d \times k, t \rangle, \langle d \times k, t \rangle \rangle \). It applies to a set of ordered pairs \( N_k \) and gives the subset of \( N_k \), such that all members of \( \pi_1(N_k) \) are plural entities with three parts each of which is an (atomic) entity in \( k \). \( \pi_2(P) \) is the context parameter on the parameterized cardinality function which is dependent on the context relative to which the count predicate has been derived. \( \langle P \rangle \) is a variable over predicates of type \( \langle d \times k, t \rangle \): \n
\[
(40) \quad [\text{Three}, \langle d \times k, t \rangle, \langle d \times k, t \rangle] = \lambda P \lambda x. x \in P \land |\pi_1(x)| \pi_2(P) = 3
\]

"*Three* denotes a function which applies to a count predicate of type \( \langle d \times k, t \rangle \) and gives the subset of the count predicate i.e. a set of ordered pairs where the first projection of each ordered pair has three parts which count as atoms in \( k \)."

**Measuring** is an operation which ignores the atomic structure of a plural entity, and assigns a value to that entity, reflecting its dimensions in terms of specified units on a dimensional scale.

\[
(41) \quad \lambda x. \text{MEAS}_D(x) = \langle n, \text{U} \rangle
\]

The structure of the scale is not context dependent, although the choice of scale used is.
What we see in *three kilos of books* is a grammatical operation which results in ignoring the atomic structure and treating *books* as a mass predicate, i.e. denoting a subset of M.

**In conclusion:**
(i) Measuring and counting classifiers express the measuring and counting operations respectively.

*three boxes of books* with the individuating reading is an expression of type $<d \times k, t>$

*three boxes/kilos of books* is an expression of type $<d, t>$.  

A mass expression denote a kind or the related set (a subset of M), which can be measured. A count expression denotes a sets of ordered pairs where the first element is an individual in $N_{root}$, a subset of M, and the second element is a context k, indicating what counts as an atom in the context, i.e. what counts as 1. They can therefore be counted.

(ii) Counting via classifiers as in *two boxes of sand, two boxes of books* allows counting of (packages of) sand/books via their containers.

(iii) Measuring cannot apply to count nouns because of a typal mismatch. Measuring of quantities of books requires ‘removing’ the atomic (and thus countable) status of the plural books in such a way that measuring can apply to it. This reflects a fundamental incompatibility between the counting and measuring operations in terms of what they do, and what they apply to.

(iv) *Books* in *two kilos/boxes of books* is a mass noun. It denotes a subset of M. It is distinguished from *book* on its Universal Grinder reading, since it contain only whole singular entities and their pluralities. But it is mass because these entities are not k-marked for the context in which they counts as atoms. (42a/b) are not equivalent).

(42) a. The dog ate 5 kilos of book
    b. The dog ate 5 kilos of books.

(v) Since container classifiers take arguments as complements (in English), we predict the infelicity of (43b) as opposed to (43a):

(43) a. I sent four boxes of (fourteen) books.
    b. #I sent four boxes of 10 kilos of books

*books* as a bare plural is a kind-denoting term and thus an argument. *Fourteen books* is likewise an argument (since we assume that the number expression raises to the determiner, and the denotation shifts to the generalised quantifier type). *10 kilos of books* is a predicate expression. *10* does not raise to determiner position, and the expression does not raise to argument type.

4. Numerical partitives in measure phrases
An outstanding issue: Why do numerical partitives occur with measure expressions?

(44) We have used up six of the three kilos of flour that I bought.

The issue: Partitives occur with both mass DPs and count DPs as in (45):
(45) Some of the furniture/pieces of furniture that I bought will be delivered this afternoon.

However, numerical partitives are restricted to count headed DPs:

(46) a. *Three of the furniture that I bought will be delivered this afternoon.
    b. Three of the pieces of furniture that I bought will be delivered this afternoon.

So if measure-classifier-expressions are mass nouns, why do numerical partitives occur with measure expressions, as in (44) above?

Rothstein (2010) shows that the restriction of numerical partitives to definites headed by count nouns follows naturally from the semantics of count expressions given above. An operation PARTITIVE(the N) recovers the set of parts of the denotation of the N. When N is a count noun, the set of parts is a set of count entities and can be counted by the number, as in three of the boys. When N is a mass noun, the set of parts is a subset of M and cannot be counted, as in #three of the furniture.

Details of Partitive analysis (Rothstein 2010):
Partitives are analysed as follows:
the is interpreted following Link 1984 in terms of the σ operation:

For Boolean algebra B: \( \sigma_B(X) = \bigcup_B(X) \) if \( \bigcup_B(X) \in X \), otherwise undefined

This applies directly to mass nouns i.e. for mass nouns the N denotes \( \sigma(N) = \sigma_m(N) \).

For count nouns the N denotes \( \sigma(N_k) = <\sigma_M(\pi_1(N_k)), k> \)

We recover the denotation of the predicate head from the DP via an operation PARTITIVE on definite DPs which gives the set of parts of \( \bigcup_N \) N, N the lexical head of DP.

The schema for the partitive operation follows the following definition schema, operating on a definite complement and giving the set of its parts:

\[
\text{PARTITIVE}(\sigma N) = \{x : x \in_M (\sigma N)\}
\]

For a mass predicate: \( \text{PARTITIVE}(\sigma(N_{\text{mass}})) = \{x : x \in_M \sigma(N_{\text{mass}})\} \), which is \( N_{\text{mass}} \) itself.

For a count predicate we lift the part-of relation on ordered pairs in \( M \times K \) from M:
\(<x_1, k> \in_k <x_2, k> \) iff \( x_1 \in_M x_2 \)

\[
\text{PARTITIVE}(\sigma(N_k)) = \{<x, k> : <x, k> \in_k <\sigma(\pi_1(N_k)), k>\}
\]

Numerical partitives occur with PARTITIVE(the N) when the set of parts of the denotation of the N is of type \( <d_k, t> \).
We correctly expect numerical partitives to occur with individuating classifier expressions as in (47): :

(47) I carried in three of the boxes of books
The set of parts of *the boxes of books* is the set of plural k-indexed box individuals, which are parts of the maximal entity in the denotation of *boxes of books*. This set is count and a numerical partitive should be possible.

But we **wrongly** predict (44), repeated here, to be ungrammatical, since *six kilos of flour* is a mass expression and the set of parts of the denotation of *the six kilos of flour* is a set in the mass domain.

(44) We have used up three of the six kilos of flour that I bought.

**Solution:** numerical measure partitives are not interpreted in the same way as numerical count partitives. In measure expressions, the number *three* in *three of the six kilos of flour* has a null complement *kilo*, which is deleted under identity with the embedded measure phrase, so (48a) and (48b) are equivalent:

(48) a. Three of the six kilos of flour that we bought have already been used up.
    b. Three kilos of the six kilos of flour that we bought have already been used up.

Support for this:
(i) ‘ordinary’ numerical partitives are impossible with mass nouns, as in (48a), but are fully grammatical when the measure head is explicit in the partitive: cf (49c) which is infelicitous

(49) a. *Two of the flour that I bought….
    b. Two kilos of the flour that I bought
    c. *Two boys of the class

(ii) The two measure expressions in (50b) need not be identical:

(50) a. 500 grams of the two kilos of fruit that I bought was rotten.
    b. 50 kilos of the 2 tons of coal that we bought was unusable.

This suggests that the higher measure head is not copied from the lower DP, but is independently generated, and may be deleted under identity with the lower measure head. (Note, this is not a copy theory of partitives. See Rothstein 2010 for general arguments against a copy theory of partitives).

This leads to the following analysis for *three(kilos) of the six kilos of flour*, with *three kilos* interpreted as a measure expression which shifts to the modifier type $<d,t>,<d,t>$:

*the six kilos of flour*: $\sigma$(SIX KILOS OF FLOUR)
*three kilos*: $\lambda P. \lambda x. x \in P \land MEAS(x) = <3,kilo>

*three kilos of the the six kilos of flour*:

$\lambda P. \lambda x. x \in P \land MEAS(x) = <3,kilo>$(PARTITIVE($\sigma$(SIX KILOS OF FLOUR))

$= \lambda P. \lambda x. x \in P \land MEAS(x) = <3,kilo> \{x: x \in_M \sigma$(SIX KILOS OF FLOUR)$\}$

$= \lambda x. x \in_M \sigma$(SIX KILOS OF FLOUR)$\} \land MEAS(x) = <3,kilo>$
References:
Rothstein,S. ms Bare noun semantics, kind interpretations, and the Universal Grinder. Ms Bar-Ilan University.